# **Evaluation of Energy Detection Spectrum Sensing based on Noise Uncertainty & Dynamic Threshold**

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Abstract—Cognitive radio is a promising technology which provides a novel way to improve utilization efficiency of available electromagnetic spectrum. Spectrum sensing is one of the most important elements in cognitive radio networks. It allows cognitive users to autonomously identify unused portions of the radio spectrum, and thus avoid interference to primary users. In this work, energy detection technique, a preferred approach for spectrum sensing in cognitive radio due to its simplicity and applicability, is considered. Energy detection sensitivity and performance drops quickly with the increment of average noise power fluctuation and becomes worse in low signal-to-noise ratio. In this paper a dynamic threshold energy detection algorithm, in which two threshold levels are used, is presented. These thresholds are used to maximize the probability of detection and minimize the probability of false alarm. Simulation results show that detection sensitivity and performance improves as the dynamic threshold factor increasing.

**Keywords:** Cognitive radio; energy detection; dynamic threshold; detectionsensitivity; noise uncertainty; probability of detection; probability of false alarm.

## 1. INTRODUCTION

Nowadays the growing demand of wireless applications has put a lot of constraints on the usage of available radio spectrum which is limited and precious resource. However, a fixed spectrum allocation has lead to underutilization of the spectrum as a great portion of licensed spectrum is not effectively utilized. Recent studies reveal that the usage of radio spectrum experiences significant fluctuations [1]. These studies conclude that heavy spectrum utilization often takes place in unlicensed bands (e.g., Industrial Scientific and Medical band, ISM), while licensed bands often experiences low (e.g., TV bands) or medium (e.g., cellular bands) utilization. This sub-optimal spectrum utilization opens new ways to spectrum access by exploiting unused spectrum bands. Cognitive radio (CR) [2] has emerged as a promising solution that can effectively address the existing conflicts between spectrum demand growth and spectrum underutilization. CR aims at improving spectrum usage efficiency by allowing

some unlicensed (secondary) users (SU) to access in an opportunistic and non-interfering manner on some licensed bands temporarily unoccupied by the licensed (Primary) users (PU).

Spectrum sensing allow SUs to autonomously detect the unused spectrum bands instantaneously and continually without the need of primary system intervention. Some popular methods of spectrum sensing are energy detection, cyclostationary detection and matched filter detection [3]. Matched filter and cyclostationary feature techniques both require prior information of PU and implementation is complex, while energy detector does not require PU information, easy to implement. Energy detector performance is very susceptible to changing noise power levels, small fluctuations in noise power may result in a sharp decline in detection performance due to SNR walls [4]. Most papers [5-8] discussed energy detection scheme based on a given constant noise power. However, the noise can be arise from various sources like quantization noise, interference between users, thermal noise, leakage of signals, etc. Therefore, it is not practical that the average noise power keeps constant in detection duration; hence the noise uncertainty is unavoidable. For those reasons, a new energy detection algorithm based on dynamic threshold is presented.

The rest of this paper is organized as follows: In section II we formulate the spectrum sensing problem. Section III reviews the classical fixed threshold energy detector & behavior under noise uncertainty. In section IV dynamic threshold algorithm is presented. Simulation results are presented in section V and the conclusions are drawn in section VI.

## 2. SPECTRUM SENSING PROBLEM FORMULATION

Assume the primary signal is independent of the noise. Spectrum sensing problem can be modeled as the binary hypothesis testing problem, where the state of the PU is defined by the following two hypothesis:

$$H_0: Y(n) = W(n)$$
(PU absent)  
$$H_1: Y(n) = X(n) + W(n)$$
(PU present) (1)

n=1, 2... N where Y(n), W(n) and X(n) corresponds to the samples of the received signal, samples of the white noise and samples of the primary signal, respectively. Noise samples W(n) are from AWGN process with variance  $\sigma_n^2$  i.e.  $W(n) \sim \mathcal{N}(0, \sigma_n^2)$  and N is number of the received signal samples collected to carry out the detection process. A missed detection occurs when a PU is present in the sensed band and the spectrum sensing algorithm selects hypothesis  $H_0$ . A false alarm occurs when the sensed band is idle and the spectrum sensing algorithm selects hypothesis  $H_1$ . So the performance of any spectrum sensing algorithm can be summarized by means of two probabilities: the probability of detection  $P_d = P_r(H_1/H_1)$  and the probability of false  $alarm P_{fa} =$  $P_r(H_1/H_0)$ . Large  $P_d$  and low  $P_{fa}$  values wood be desirable for good CR performance.

#### 3. SYSTEM MODEL

#### 3.1 Fixed threshold energy detector

If we have knowledge about the average power of the signal X(n) only then energy detector is the best choice for optimal detection, the test statistics is given by

$$D(Y) = \frac{1}{N} \sum_{n=0}^{N-1} Y^2(n) \underset{H_0}{\overset{H_1}{\geq}} \gamma$$
(2)

Where D(Y) is the decision variable and  $\gamma$  is the decision threshold. The test statistics follows a central (under hypothesis  $H_0$ ) and non-central (under hypothesis  $H_1$ ) chisquare distribution with N degrees of freedom [9]. In low SNR regimes, the number of samples used for good detection is large enough (N >> 1). So we can make use of central limit theorem to approximate the test statistics as Gaussian distribution as follows: (provided the noise variance is known and noise uncertainty is null) [4] [10].

$$D(Y) \sim \begin{cases} \mathcal{N}(\sigma_n^2, 2\sigma_n^4/N) & H_0 \\ \mathcal{N}(P + \sigma_n^2, 2(P + \sigma_n^2)^2/N) & H_1 \end{cases}$$
(3)

Where P is the average signal power,  $\sigma_n^2$  is the noise variance. If only AWGN noise is considered, then, we can obtain the detection probability  $P_D$ , false alarm probability  $P_{FA}$  and missing probability  $P_{MD}$ , respectively [4] [10]:

$$P_{D} = P_{r}(D(Y) > \gamma | H_{1}) = Q\left(\frac{\gamma - (P + \sigma_{n}^{2})}{\sqrt{2/N} (P + \sigma_{n}^{2})}\right)(4) \qquad P_{FA} = \sigma^{2} \epsilon \left[\sigma_{n}^{2} / \rho, \rho \sigma_{n}^{2}\right] Q\left(\frac{\gamma - \sigma^{2}}{\sqrt{2/N} \sigma_{n}^{2}}\right) = Q\left(\frac{\gamma - \rho \sigma_{n}^{2}}{\sqrt{2/N} \rho \sigma_{n}^{2}}\right)(9) \\ P_{FA} = P_{r}(D(Y) > \gamma | H_{0}) = Q\left(\frac{\gamma - \sigma_{n}^{2}}{\sqrt{2/N} \sigma_{n}^{2}}\right)(5) \qquad P_{D} = \min_{\sigma^{2} \epsilon \left[\sigma_{n}^{2} / \rho, \rho \sigma_{n}^{2}\right]} Q\left(\frac{\gamma - (P + \sigma^{2})}{\sqrt{2/N} (P + \sigma^{2})}\right) \\ P_{MD} = 1 - P_{D} = 1 - Q\left(\frac{\gamma - (P + \sigma_{n}^{2})}{\sqrt{2/N} (P + \sigma_{n}^{2})}\right)(6) \qquad = Q\left(\frac{\gamma - (P + \sigma_{n}^{2} / \rho)}{\sqrt{2/N} (P + \sigma_{n}^{2} / \rho)}\right)(10)$$

Where Q(.) is the standard Gaussian complementary cumulative distribution function (CDF). Computing  $\gamma$  in terms of  $P_D \& P_{FA}$  from (5) we get,

$$\gamma = \sigma_n^2 \left( 1 + Q^{-1}(P_{FA}) \sqrt{2/N} \right) (7)$$

 $Q^{-1}(.)$  is the inverse Where standard Gaussian complementary CDF.N is expressed in terms of  $P_D$ ,  $P_{FA}$  and SNR from (4), (5)

$$N = 2[Q^{-1}(P_{FA}) - Q^{-1}(P_D)(1 + SNR)]^2 SNR^{-2}$$
(8)

SNR= $P/\sigma_n^2$  is the signal to noise ratio. It may note that the expression of N in (7) is free from variable  $\gamma$ (decision threshold). This shows that if the noise power  $\sigma_n^2$  were completely known, then signals could be detected at arbitrarily low SNRs by increasing the sensing time N.

#### 3.2 Noise uncertainty

We have discussed and analyzed the case without noise uncertainty. Now we take the noise uncertainty into account. The distributional uncertainty of noise can be included in a single interval  $\sigma^2 \epsilon [\sigma_n^2 / \rho, \rho \sigma_n^2]$  as shown in



Fig. 1: Noise uncertainty for energy detector

fig. 1, where  $\rho$  is the noise uncertainty factor and the value of  $\rho$ is closer to 1, that is  $\rho > 1$  and  $\rho \approx 1$ . Thus (4) and (5) are modified to get

Eliminating  $\gamma$  and it has:

$$N = \frac{2[\rho Q^{-1}(P_{FA}) - (1/\rho + SNR)Q^{-1}(P_D)]^2}{(SNR - (\rho - 1/\rho))^2}$$
(11)

Energy detection algorithm with fixed threshold offers degraded performance with noise uncertainty. This indicates that the choice of a fixed threshold is no longer valid under noise uncertainty and threshold should be chosen flexible. In next section, energy detection algorithm with dynamic threshold is presented.

## 4. DYNAMIC THRESHOLD ALGORITHM

Because of noise uncertainty, performance decline quickly and introduces cognitive user interference to the licensed users. For this reason, we present a dynamic threshold algorithm here.

Assuming  $\rho'$  is the dynamic threshold factor and  $\rho' > 1$ and  $\rho' \approx 1$ .  $\rho'$  is introducing in such a way that threshold  $\gamma$  lie in the interval  $[\gamma/\rho', \rho'\gamma]$ , instead of remaining constant [11]. Here, we will consider the noise uncertainty and dynamic threshold respectively. From (9) and (10), it has:

$$P_{D} = \frac{\min}{\gamma' \epsilon} \left[ \frac{\gamma}{\rho'}, \rho' \gamma \right] \sigma^{2} \epsilon \left[ \frac{\sigma_{n}^{2}}{\rho}, \rho \sigma_{n}^{2} \right] Q \left( \frac{\gamma' - (P + \sigma^{2})}{\sqrt{2/N} (P + \sigma^{2})} \right)$$
$$= Q \left( \frac{\gamma/\rho' - (P + \sigma_{n}^{2}/\rho)}{\sqrt{2/N} (P + \sigma_{n}^{2}/\rho)} \right) (12)$$
$$P_{FA} = \gamma' \epsilon \left[ \frac{\gamma}{\rho'}, \rho' \gamma \right] \sigma^{2} \epsilon \left[ \frac{\sigma_{n}^{2}}{\rho}, \rho \sigma_{n}^{2} \right] Q \left( \frac{\gamma' - \sigma^{2}}{\sqrt{2/N} \sigma^{2}} \right)$$
$$= Q \left( \frac{\rho' \gamma - \rho \sigma_{n}^{2}}{\sqrt{2/N} \rho \sigma_{n}^{2}} \right) (13)$$

Eliminating  $\gamma$ , and computing the value of N in terms of  $P_D$ ,  $P_{FA}$ ,  $\rho$ ,  $\rho'$  and

SNRN = 
$$2 \frac{[(\rho/\rho')Q^{-1}(P_{FA}) - \rho'(1/\rho + SNR)Q^{-1}(P_D)]^2}{(\rho'SNR + \rho'/\rho - \rho/\rho')^2}$$
 (14)

#### 5. SIMULATION RESULTS

Let us investigate carefully above formulations through MATLAB simulations to have deeper illustrative insight into different aspects of spectrum sensing technique.

#### 5.1 Without noise uncertainty

The probability of false alarm  $P_{FA} \in (0, 1.0)$  has been set to a minimum value of 0.01 which may be accepted as negligible. In Fig. 2, four sample points with values N=300, 1000, 2500 and 5000 over the SNR range of -25 to 5 dB are considered. It shows that for a fixed value of SNR the probability of detection is improved by increasing the number of samples. It is also, observed in fig. 2 that as N increases, there is a significant increase in  $P_D$  even at low values of SNR.



Fig. 2: *P<sub>D</sub>*Vs. SNR for different values of N

Fig. 3 shows the curves which provides the information about  $P_D$  vs.  $P_{FA}$  for different values of SNR and N=1000. From the graph it is inferred that i.) For SNR values at -20 dB, the  $P_D$  becomes almost equal to  $P_{FA}$  which depicts that energy detector becomes non-robust under low SNR. ii.) The performance becomes better at higher SNR values as the  $P_D$  increases.

#### 5.2 With noise uncertainty

When  $\rho \approx 1$ , then  $SNR^{-2} \approx (SNR - (\rho - 1/\rho))^{-2}$ , the numerical value of (11) and (8) are almost the same; When  $\rho$  is larger and suppose  $\rho = 1.05$ , then  $(\rho - 1/\rho) = 0.0976 \approx 0.1$ , if SNR= 0.1, well then  $(SNR - (\rho - 1/\rho))^{-2} \approx 0$ , substituting into equation (11) to be N $\rightarrow \infty$ . In other words only an infinite detection duration can complete detection, which is impracticable.

A tiny fluctuation of average noise power causes performance drop seriously, especially with a lower SNR. Fig. 4 is the numerical results of (11) given: SNR= -10dB,  $P_{FA} \in (0,0.05)$  and N = 1000.

In Fig. 4  $\rho$ =1.00 represents no noise uncertainty. We can see that the performance gradually drops as the noise uncertainty

CONTRACTOR OF 0.9 0.8 0.7 Probability of detection(Pd) 0.6 0.5 0.4 0.34 0.20 SNR= -20 Π - SNR= -15 -SNR= -10 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 Probability of false alarm(Pfa)

Fig. 3: P<sub>D</sub>Vs. P<sub>FA</sub> for different values of SNR





For example, if  $P_{FA}$ =0.1, then  $P_D$ < 0.15, even when  $P_{FA}$ =0.5,  $P_D$  is still less than 50%. It means that rental users decide the spectrum is idle no matter whether there are primary users present. This indicates that Energy detector is very sensitive to noise uncertainty. In order to guarantee a good performance, dynamic threshold is chosen.

# 5.3 With Dynamic Threshold

In (14), when  $\rho' \approx \rho \operatorname{and} \rho' / \rho \approx \rho / \rho' \approx 1$ , then  $(\rho'SNR + \rho' / \rho - \rho / \rho')^{-2} \approx (SNR)^{-2}$  and  $\rho'(1/\rho + SNR) \approx (1 + SNR)$ , the numerical value of (14) is almost the same to (8). Therefore, dynamic threshold detection algorithm can

overcome the noise uncertainty as long as a suitable dynamic threshold factor is chosen. Comparing (14) with (11), let SNR= 0.1 and  $\rho' = \rho \approx 1$ , it is clear that  $(\rho'SNR + \rho'/\rho - \rho/\rho')^{-2} \gg (SNR - (\rho - 1/\rho))^{-2}$ . Therefore, to achieve the same detection performance, the detection duration N of dynamic threshold detection scheme is significantly shorter. Fig. 5 is the numerical results of (8), (11) and (14). With the same parameters as before.

Where  $\rho = 1.00$  denotes that the average noise power keeps constant means without noise uncertainty;  $\rho' = 1.00$  denotes that the algorithm did not use dynamic threshold means the threshold is fixed; otherwise, it represents cases with noise uncertainty and dynamic threshold. It is shown in Fig. 5 that the dynamic threshold makes the performance more accurate as the dynamic threshold is increasing. This method significantly increases the robustness against noise average power fluctuation without increasing the detection duration.



Fig. 5: ROC curves of energy detection scheme with different  $\rho$  and  $\rho'$ 

## 6. CONCLUSION

In this paper, the relationship of energy detection performance with detection duration, detection sensitivity and noise average power fluctuation is analyzed. A fractional fluctuation of average noise power will lead to the quick drop of spectrum detection performance. So to overcome this drawback, a new energy detection algorithm based on dynamic threshold is presented. This algorithm leads to an accurate detection performance even if there is a serious noise uncertainty in the case of low SNR. This scheme can enhance the robustness of combatting noise uncertainty and improve the capacity of spectrum sensing. How to acquire the practical noise uncertainty factor and detection threshold is our future work.

factor increasing. When  $\rho$ =1.05, the performance dropped seriously.

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